Angularities and other shapes

Lorenzo Magnea
Dipartimento di Fisica Teorica, Università di Torino and INFN, Sezione di Torino,
Via P. Giuria, 1, I–10125 Torino, Italy

I discuss soft-gluon resummation and power corrections for event shape distributions, mostly in e^+e^- annihilation. I consider specifically the thrust, the C parameter, and the class of angularities, and show how factorization techniques and dressed gluon exponentiation lead to predictive models of power corrections that are firmly grounded in perturbative QCD. The scaling rule for the shape function for angularities is derived as an example. Finally, I make a few remarks on possible generalizations to hadron collisions, and on their relevance to LHC studies.

1. INTRODUCTION

Quantum Chromodynamics is made special among phenomenologically relevant field theories by the property of confinement. Given that the lagrangian is written in terms of fields that do not appear in the construction of the true asymptotic states of the theory, it may seem surprising that perturbative calculations performed around the trivial vacuum have any relevance at all. The predictive power of perturbative QCD, in the presence of a kinematic scale Q^2 much larger than the confinement scale Λ^2, is rescued by asymptotic freedom, combined with quantum-mechanical incoherence and gauge invariance. These are the necessary ingredients entering the proof of factorization theorems [1], which are the cornerstones of all PQCD calculations.

Factorization theorems in essence provide a bound for the parametric size of nonperturbative corrections to high energy inclusive cross sections. Such corrections are typically suppressed by powers of the hard scale Q^2. It should be emphasized, however, that factorization theorems are proven perturbatively, by examining the all-order structure of long-distance singularities in Feynman diagrams. Their phenomenological relevance must then rely upon the additional (if plausible) assumption that confinement be a relatively soft process, happening without a violent rearrengement of momentum configurations, as colored particles evolve away from the hard scattering event. Decades of experience with QCD phenomenology have taught us that this assumption is very well borne out by the data. Indeed, granted this assumption, the bound on the size of nonperturbative effects provided by the factorization theorem is the first and simplest case of nonperturbative information extracted form QCD by purely perturbative methods.

The general idea that perturbation theory, whenever genuine all-order information is available, can provide important clues to understand nonperturbative effects, has subsequently been applied successfully in a variety of situations. Typically, the perturbative expansion is found to diverge, and the uncertainty in the physical prediction due to this divergence is interpreted as a measure of the size of the expected nonperturbative correction. Specifically, the nonperturbative contribution must be ambiguous by an amount matching the uncertainty in the perturbative prediction. The assumption that the actual size of the nonperturbative corrections should be well represented by this ambiguity is sometimes referred to as ultraviolet dominance of power corrections [2].

The all-order perturbative information required to begin any study of power corrections has mostly been provided by two complementary sources: renormalon-type calculations (reviewed in [3]), which roughly speaking target running-coupling effects by summing up fermion bubble corrections to single gluon emission, and soft gluon resummations, which make use factorization and universality to compute leading multigluon contributions in the soft and collinear limits (for a recent review, see [4]). Recently, it was shown that the two appoaches can be combined [5], yielding a strongly constrained and rather elegant model of power corrections in the Sudakov region.

Event shape distributions in hard collisions are an especially interesting class of observables for power correction studies, and indeed a lot work has been done in the past several years on the subject, especially in the context of e^+e^- annihilation and DIS [6, 7]. Event shape distributions, in fact, provide a continuous interpolation between processes featuring mostly hard, perturbative radiation and configurations dominated by soft and collinear gluon emission. The corresponding theoretical prediction must then be constructed matching a variety of tools: NLO perturbative results for hard emissions, soft gluon resummations when the value of the event shape forces radiation to be soft, and finally models of power corrections very close to threshold.

In general, models of power corrections involve nonperturbative parameters or functions, which must be determined from experiment, much as one does with parton distributions. The predictive power of these models must then rely on a degree of universality of soft radiation, which is well understood in perturbation theory, and must be assumend to hold nonperturbatively as well. By comparing theoretical predictions for different but related event shapes one can then test our understanding of QCD at or beyond the strict limits of applicability of perturbation theory.

The application of these techniques has lead in recent years to quantitatively testable and quite successful models of power corrections. Here I will mostly discuss results obtained by Dressed Gluon Exponentiation (DGE) [5], as applied to thrust [8, 9], the C-parameter [10], and the class of angularities [11, 12]. These examples show that current tools lead to simple, analytical, quantitative results that can readily be compared with experimental data. Most strikingly, leading power corrections to angularity distributions obey a simple scaling rule as a function of a continuous parameter, which gives a powerful test of our understanding of soft QCD in electron-positron annihilation. In Sect. (2) I will briefly summarize the formalism of shape functions for event shape distributions, and show how DGE provides a renormalon model for shape functions incorporating the constraints of NLL soft gluon resummation. I will use mostly thrust as a working example, comparing at the end with similar results obtained for the C parameter. In Sect. (3) I will discuss the class of angularities and derive the scaling rule, and finally in Sect. (4) I will briefly comment on possible extensions of these techniques to hadron-hadron collisions.

2. SOFT GLUON EFFECTS FOR EVENT SHAPE DISTRIBUTIONS

An event shape distribution is a weighted cross section, assigning a prescribed value to a specific infrared and collinear safe combination of the momenta of final state particles in a high energy collision. In the case of e^+e^- annihilation, let $F_m(p_1, \ldots, p_m)$ be one such combination, computed for an m-parton final state. The distribution of the associated event shape f is then

$$\frac{d\sigma}{df} = \frac{1}{2Q^2} \sum_m \int d\mathrm{LIPS}_m \; \overline{|\mathcal{M}_m|^2} \; \delta \left(f - F_m(p_1, \ldots, p_m) \right) \;, \tag{1}$$

where $\mathcal{M}_m$ is the appropriate matrix element. In the following, I will consider event shapes f which vanish in the limit of a pencil-like two-jet event. A prime and well-known example is $\tau = 1 - T$, with T the thrust,

$$T = \max_{\hat{n}} \left[\frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{Q} \right] \;, \tag{2}$$

which I will use below to illustrate the general features of the approach. Other shapes I will consider include the C-parameter,

$$C = 3 - \frac{3}{2} \sum_{i,j} \frac{(p_i \cdot p_j)^2}{(p_i \cdot Q)\,(p_j \cdot Q)} \;, \tag{3}$$

which does not require a maximization procedure, and the one-parameter class of angularities,

$$\tau_a = \frac{1}{Q} \sum_i (p_\perp)_i \mathrm{e}^{-|\eta_i|(1-a)} \;. \tag{4}$$

where rapidity η_i and transverse momentum $p_{\perp,i}$ are computed with respect to the thrust axis, so that for $a = 0$ one verifies that $\tau_0 = \tau$.

The common feature of these event shapes, which opens the way to an all-order perturbative analysis and to studies of power corrections, is the fact that for small values of f all radiation is constrained to be soft or collinear. As a consequence, the distributions develop double logarithmic singularities of Sudakov type, which can (and must) be resummed thanks to the universal properties of soft radiation and to the factorizability of the cross section in the Sudakov limit. In QCD, resummation displays the ambiguity of perturbation theory, originating from the presence of the running coupling evaluated at soft scales. This leads naturally to models of power corrections. I will now illustrate the general features of the method using thrust as an example.

2.1. Resummation

In order to resum singular contributions to the thrust distribution in the limit $\tau \to 0$, one needs to take a Laplace transform, which factorizes the δ-function constraint fixing the value of τ. Logarithmic contributions then exponentiate according to

$$\int_0^\infty d\tau e^{-\nu\tau} \frac{1}{\sigma}\frac{d\sigma}{d\tau} = \exp\left[\int_0^1 \frac{du}{u}\left(e^{-u\nu}-1\right)\left(B\left(\alpha_s\left(uQ^2\right)\right) + 2\int_{u^2Q^2}^{uQ^2}\frac{dq^2}{q^2}A\left(\alpha_s(q^2)\right)\right)\right]. \tag{5}$$

The pattern of exponentiation is highly nontrivial, since the double logarithms of the ordinary perturbative expansion turn into single logarithms in the exponent. Generically one finds a structure of the form [13]

$$\sum_k \alpha_s^k \sum_p^{2k} c_{kp} L^p \to \exp\left[L\, g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right], \tag{6}$$

where L is the logarithm of the transformed variable, $L = \log\nu$ in this case. Leading logarithms (LL) are generated to all orders by the function g_1, which is completely determined by the knowledge of the anomalous dimension $A(\alpha_s)$ to one loop. Next-to-leading logarithms (NLL), corresponding to the function g_2, require the knowledge of $A(\alpha_s)$ to two loops and $B(\alpha_s)$ to one loop. NLL accuracy is the common standard for resummation of event shape distributions. Note however that Sudakov resummation, expressed here by Eq. (5), in general involves one more function, $D\left(\alpha_s(u^2Q^2)\right)$. This function is associated with wide-angle soft gluon emission, and is process-dependent, unlike the anomalous dimension A. To any finite logarithmic accuracy, the contributions of D can be reproduced by modifying B in a process-dependent manner, and for the event shapes discussed here D does not give any contribution at NLL level. The fact that D dpends on the scale uQ, however, has important consequences on power corrections, as discussed below.

2.2. Power Corrections

One can deduce from Eq. (5) (where the integration variable u in the exponent plays the role of τ) that for small values of τ there are two relevant momentum scales: τQ^2 and $\tau^2 Q^2$. This can be understood from the physical picture underlying Sudakov factorization: at small τ gluon radiation can be organized into jets of particles collinear to the primary partons, with invariant mass proportional to $\sqrt{\tau}Q$, plus the contribution of wide-angle soft gluons, characterized by their total energy τQ. It is natural to expect that power corrections will be organized by these two scales, and thus be of the form $(\Lambda^2/(\tau Q^2))^m$ and $(\Lambda^2/(\tau^2 Q^2))^n$ respectively. Clearly, when $\tau \sim \Lambda/Q$ all power corrections of this second kind become important, and must be collectively taken into account. Power corrections in the larger scale, on the other hand, become important only when $\tau \sim \Lambda^2/Q^2$, a value which is too small to be relevant for LEP fits. The need for power corrections is apparent in Eq. (5), since the integrals over momentum scales are perturbatively ill-defined because of the Landau singularity of the running coupling. An elegant way to summarize the nonperturbative information encoded in Eq. (5) was described in [14]. The basic assumptions are the applicability of the factorization underlying Eq. (5) all the way down to values of τ such that $\Lambda^2 \sim \tau^2 Q^2 \ll \tau Q^2 \ll Q^2$, and the existence of a nonperturbative definition of the running coupling rendering the scale integrals well defined. Consider then, for example, the term in Eq. (5) containing the anomalous dimension A. In order to disentangle perturbative and nonperturbative domains, one can simply introduce a factorization scale μ, switch the order of the q^2 and u integrations, and define

$$S(\nu, Q^2) \equiv \int_0^1 \frac{du}{u} \left(\mathrm{e}^{-u\nu} - 1\right) \int_{u^2Q^2}^{uQ^2} \frac{dq^2}{q^2} A\left(\alpha_s(q^2)\right) = S_{\mathrm{NP}}(\nu/Q, \mu)\, S_{\mathrm{PT}}(\nu, Q, \mu) \,,$$

$$S_{\mathrm{NP}}(\nu/Q, \mu) \equiv \int_0^{\mu^2} \frac{dq^2}{q^2} A\left(\alpha_s(q^2)\right) \int_{q^2/Q^2}^{q/Q} \frac{du}{u} \left(\mathrm{e}^{-u\nu} - 1\right) = \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{\nu}{Q}\right)^n \lambda_n(\mu^2) + \mathcal{O}\left(\frac{\nu}{Q^2}\right) . \tag{7}$$

The last equality expresses a set of nonperturbative contributions to the Sudakov exponent in terms of moments of the anomalous dimension A at low scales. These moments,

$$\lambda_n(\mu^2) = \frac{1}{n} \int_0^{\mu^2} dq^2\, q^{n-2} A\left(\alpha_s(q^2)\right) \,, \tag{8}$$

are not computable in perturbation theory: much like parton distributions, they should be measured for a given observable at a given factorization scale, and then used to predict different observables, based on their universality properties. In general, the full set of leading nonperturbative corrections will involve also moments of the function D, which also parametrize power corrections of the form $(\Lambda\nu/Q)^n$. These corrections thus have a universal component, expressed in terms of the anomalous dimension A, and a process-dependent component given by the function D. At power accuracy, it is natural to disentangle the contributions of B and D by requiring that the function B should be the same appearing in the resummation formula for DIS structure functions, where the corresponding D function is known to vanish [15]. Expressions like Eq. (7) provide a framework to test universality, or to construct specific models of power corrections. To summarize the effects of the parameters $\lambda_n(\mu^2)$ one can use them to build up a "shape function", according to

$$\exp\left[S_{\mathrm{NP}}(\nu/Q, \mu)\right] \equiv \int_0^{\infty} d\epsilon\, \mathrm{e}^{-\nu\epsilon/Q}\, f_{\tau,\mathrm{NP}}(\epsilon, \mu) \,. \tag{9}$$

Here ϵ can be interpreted as the total energy carried into the final state by soft gluons at scales below μ. Confining oneself to the leading power correction, corresponding to the first moment $\lambda_1(\mu^2)$, one recovers the result of the "tube model" [16]: that nonperturbative effects shift the distribution away from the small τ region by an amount proportional to the average energy carried away by soft radiation. Subleading moments provide additional smearing.

2.3. Dressed gluon exponentiation

The shape function idea is very general, and can be used both to test universality, by connecting power corrections to related event shapes [17], or to construct models based on factorization and Lorentz invariance in specific cases [18]. One can get more detailed predictions by making stronger assumptions: for example, one can apply a renormalon model, and study the corresponding power corrections in the Sudakov region. This is the basic idea underlying dressed gluon exponentiation (DGE) [5]. I will now summarize the basic steps of this method, using thrust as an example.

First of all, one computes the single gluon contribution to the event shape under study, for a gluon of nonvanishing virtuality $\xi = k^2/Q^2$. This is the characteristic function of the dispersive approach [19, 20] to power corrections. Since we are interested in the Sudakov region, we need to retain only terms that are singular as $\tau \to 0$. Given the characteristic function $\mathcal{F}(\xi, \tau)$, one can write a clean representation of the single gluon cross section by introducing a Borel representation for the strong coupling. One defines

$$\bar{A}(\xi Q^2) = \int_0^{\infty} du\, \xi^{-u} \left(Q^2/\Lambda^2\right)^{-u} \frac{\sin \pi u}{\pi u}\, \mathrm{e}^{\kappa u} \,. \tag{10}$$

This amounts to an analytic continuation of the strong coupling at the scale k^2 from the euclidean to the timelike region, formally valid in the large-n_f limit. The factor $\mathrm{e}^{\kappa u}$ is renormalization-scheme dependent, with $\kappa = 5/3$ in the

$\overline{MS}$ scheme. The single dressed gluon cross section is then

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau}(\tau,Q^2) = -\frac{C_F}{2\beta_0}\int_0^1 d\xi\,\frac{d\mathcal{F}(\xi,\tau)}{d\xi}\,\bar{A}(\xi Q^2) \;\equiv\; \frac{C_F}{2\beta_0}\int_0^\infty du\,\left(Q^2/\Lambda^2\right)^{-u}\,B(\tau,u)\,, \tag{11}$$

where in the last equality I introduced the Borel function $B(\tau,u)$, which contains the physical information on the thrust distribution. The strategy of performing all integrals except the one over the Borel parameter yields at the end a trasparent representation for power corrections. In the case of thrust, the terms responsible for logarithmic enhancements in $\dot{\mathcal{F}}(\xi,\tau)$ are given by

$$\dot{\mathcal{F}}(\xi,\tau)\Big|_{\log} = 2\left(\frac{2}{\tau}-\frac{\xi}{\tau^2}-\frac{\xi^2}{\tau^3}\right)\,, \tag{12}$$

which gives a Borel function of the form

$$B(\tau,u)|_{\log} \;=\; 2\,\mathrm{e}^{\kappa u}\,\frac{\sin\pi u}{\pi u}\left[\frac{2}{u}\,\tau^{-1-2u}-\tau^{-1-u}\left(\frac{2}{u}+\frac{1}{1-u}+\frac{1}{2-u}\right)\right]. \tag{13}$$

Note that already at this stage one can make several useful observations. Poles in $B(\tau,u)$ at, say, $u=u_0$, would correspond to renormalon singularities in the distribution, and expected power corrections of size $(\Lambda/Q)^{2u_0}$; in fact, $B(\tau,u)$ has no such poles: the would-be singularities at $u=1,2$ are cancelled by the factor $\sin\pi u$, and poles at $u=0$ cancel between the two terms in square brackets, because of the infrared safety of thrust. Renormalons arise when taking moments of the distribution, because of the convergence constraints on the Borel integral at small values of τ. One also notes that the first term in Eq. (13) is associated with soft wide-angle radiation, since it generates in Eq. (11) terms proportional to $(\Lambda/(\tau Q))^{2u}$. Similarly, the second term in Eq. (13) is associated with the jet function, contaning collinear as well as soft enhancements.

The key step in DGE is to note that at LL level Sudakov resummation yields a simple exponentiation of the one-gluon emission cross section in moment space. One can then retain all large-n_f information, and the corresponding model of power corrections, in the Sudakov exponent by simply using the single dressed gluon cross section as kernel of exponentiation. One defines

$$\left(\frac{1}{\sigma}\frac{d\sigma}{d\tau}\right)_{\rm DGE} = \int_{k-\mathrm{i}\infty}^{k+\mathrm{i}\infty}\frac{d\nu}{2\pi\mathrm{i}}\,\mathrm{e}^{\nu\tau}\,\exp\left[-E(\nu,Q^2)\right]\,, \tag{14}$$

where the Sudakov exponent is now given by

$$E(\nu,Q^2) \;=\; \int_0^\infty d\tau\,\left(1-\mathrm{e}^{-\nu\tau}\right)\left(\frac{1}{\sigma}\frac{d\sigma}{d\tau}\right)_{\rm SDG} \equiv \frac{C_F}{2\beta_0}\int_0^\infty du\,\left(Q^2/\Lambda^2\right)^{-u}\,B_\tau(\nu,u)\,. \tag{15}$$

Here the single dressed gluon cross section is defined by Eq. (11), virtual corrections have been taken into account by subtracting the value of the Laplace transform at $\nu=0$, and in the second equality the Borel function $B_\tau(\nu,u)$ for the Sudakov exponent has been defined. As usual, all integrals are performed except the one on the Borel parameter u. Although formally similar to Eq. (11) for the single dressed gluon cross section, Eq. (15) has a much richer physical content, displayed by the nontrivial renormalon structure of the Borel function $B_\tau(\nu,u)$. This is a consequence of the fact that exponentiation, subject to the constraint of energy conservation, has promoted the single gluon result to a genuine approximation for multigluon emission[1]. In the specific case of the thrust, the result for the Borel function is

$$B_\tau(\nu,u) \;=\; 2\,\mathrm{e}^{\kappa u}\,\frac{\sin\pi u}{\pi u}\left[\Gamma(-2u)\left(\nu^{2u}-1\right)\frac{2}{u}-\Gamma(-u)\left(\nu^u-1\right)\left(\frac{2}{u}+\frac{1}{1-u}+\frac{1}{2-u}\right)\right]. \tag{16}$$

[1] In fact, the exponent $E(\nu,Q^2)$ has a natural interpretation in terms of the Borel representation of Sudakov anomalous dimensions associated respectively with soft and collinear radiation [15]. Corrections subleading in n_f change the the logarithmic behavior of the cross section as well as the size of the residues of poles in the Borel plane, but they are not expected to modify the analytic structure of $B_\tau(\nu,u)$, which determines which power corrections are actually present.

It is useful to compare this result with the one for the C-parameter [10]

$$B_c(\nu,u) \, = \, 2\, \mathrm{e}^{\kappa u}\, \frac{\sin \pi u}{\pi u} \, \left[\Gamma(-2u) \left(\nu^{2u} - 1\right) 2^{1-2u} \frac{\sqrt{\pi}\Gamma(u)}{\Gamma(\frac{1}{2}+u)} \, - \, \Gamma(-u)\left(\nu^{u} - 1\right) \left(\frac{2}{u} + \frac{1}{1-u} + \frac{1}{2-u} \right) \right] \, . \tag{17}$$

Clearly, Eqs. (16) and (17) are very similar; there are, however, important differences, which highlight the degree of universality to be expected in comparing the two distributions, and which can in principle be tested experimentally. First of all, the Borel functions contain perturbative information on Sudakov logarithms: although the exponentiation was performed assuming independent gluon radiation, it can be shown [8] that one can upgrade the formalism to NLL accuracy by simply replacing the running coupling Eq. (10) with the two-loop expression, and by changing renormalization scheme, including in the constant κ the contribution of terms singular as $x \to 1$ in the NLO Altarelli-Parisi splitting function (the "gluon bremsstrahlung" scheme [21]). Beyond NLL, the coefficients of all subleading logarithms can be computed in the large n_f limit, by simply expanding the Borel function in powers of u, and replacing $u^n \to n!(b_0 \alpha_s/\pi)^{n+1}$. Computing subleading logarithms uncovers the factorial growth of their coefficients, and can be used to gauge the reliability of perturbative resummation in different kinematical regimes. Next, one may observe that the infrared safety of τ and C is once again reflected in the cancellation of the poles of both Borel functions at $u = 0$. One also notes that wide-angle soft radiation and collinear gluons contribute as before two separate terms, and the jet functions (the terms proportional to $\Gamma(-u)$) are identical for the two observables[2]. They contribute renormalons at $u = 1, 2$, corresponding to exponentiated power corrections of the form $(\Lambda^2 \nu/Q^2)^{1,2}$. Soft gluon contributions, on the other hand, have renormalons at $u = m/2$, for all odd values of m, yielding the leading power corrections $(\Lambda \nu/Q)^m$, and they are quantitatively different for τ and C, distinguishing the two observables. Specifically, by taking the ratio of the two "soft functions" (the terms proportional to $\Gamma(-2u)$) and expanding in powers of u, one can verify that the two observables begin to differ perturbatively at NNLL level, as predicted in [22], but the growth of the coefficients of further subleading logarithms is weaker for the C-parameter than for the thrust. Similarly, if one boldly takes the large-n_f residues of the poles of the Borel functions as a reasonable estimate of the size of the corresponding power corrections, one observes that $(\Lambda \nu/Q)^m$ corrections are systematically smaller for the C parameter: the two shape functions should therefore differ, and one expects that the resummed perturbative prediction, as well as the approximation of the shape function by a constant shift, should work better phenomenologically for C than for τ.

3. THE CLASS OF ANGULARITIES

The discussion in Sect. (2) illustrates the predictive power of DGE. Another interesting application concerns angularities, whose definition, Eq. (4), can be rewritten for massless particles as

$$\tau_a \, = \, \frac{1}{Q} \sum_i \omega_i \left(\sin\theta_i\right)^a \left(1 - \left|\cos\theta_i\right|\right)^{1-a} \quad , \tag{18}$$

with θ_i the angle with respect to the thrust axis. Angularities (so christened in [12]) were introduced in [23] as auxiliary shape variables used to tame nonglobal logarithms [24] for observables related to out-of-jet energy flow. They have several remarkable features, which make them very interesting for our understanding of QCD at the edge of the perturbative domain . First of all, they are characterized by a tunable parameter a, which can be used to interpolate between different shapes, or to bring to focus specific momentum configurations in a continuous way. The parameter a must satisfy $a < 2$ for infrared safety, and a tighter restriction $a < 1$ is required in order to preserve a relatively simple resummation in the Sudakov region, in the form of Eq. (5): for $1 \leq a < 2$ further logarithmic singularities associated with jet recoil must be taken into account [25]. For $a = 1$, one recognizes that $\tau_1 = B$, the

[2]Note that $B_c(\nu, u)$ is computed for a rescaled C parameter, $c = C/6$

broadening; for $a = 0$, $\tau_0 = 1 - T$; for negative a, events dominated by high rapidity give increasingly suppressed contributions to τ_a, which in turn suppresses power corrections of collinear origin; finally, for $a \to -\infty$ the distribution becomes a δ function at $\tau_a = 0$, with a strenght given by the total cross section.

Remarkably, although the relative weights of rapidity and transverse momentum change with a, it is possible to derive a resummation formula [23] of the form of Eq. (5), valid for $a < 1$. Indeed, at NLL accuracy one can write

$$\ln\left[\tilde{\sigma}\left(\nu, a\right)\right] \; = \; \int_0^1 \frac{du}{u} \left[B\left(\alpha_s(uQ^2)\right) \left(\mathrm{e}^{-u\,\nu^{2/(2-a)}} - 1\right) \, + \, 2 \int_{u^2Q^2}^{uQ^2} \frac{dq^2}{q^2} \, A\left(\alpha_s(q^2)\right) \left(\mathrm{e}^{-u^{1-a}\nu (q/Q)^a} - 1\right) \right] . \tag{19}$$

The a dependence of Sudakov logarithms is clearly nontrivial: as an example, the function $g_1(\alpha_s L)$ responsible for leading logarithms in Eq. (6) is given by

$$g_1(x, a) \; = \; - \, \frac{4}{\beta_0} \, \frac{2-a}{1-a} \, \frac{A^{(1)}}{x} \left[\frac{1-x}{2-a} \, \ln\left(1-x\right) \, - \, \left(1 - \frac{x}{2-a}\right) \ln\left(1 - \frac{x}{2-a}\right) \right] . \tag{20}$$

Power corrections, however, turn out to have a much simpler a dependence [11]. Performing the analysis leading to Eq. (8), one easily finds that in the nonperturbative region all moments of the anomalous dimension A are multiplied by a simple common factor

$$\lambda_n^{(a)}(\mu^2) \; = \; \frac{1}{1-a} \, \frac{1}{n} \int_0^{\mu^2} dq^2 \, q^{n-2} A\left(\alpha_s(q^2)\right) \; = \; \frac{1}{1-a} \, \lambda_n^{(0)}(\mu^2) \; . \tag{21}$$

As a consequence, the Laplace transform of the shape function defined by Eq. (9) obeys a simple and remarkable scaling rule

$$\tilde{f}_{a,\mathrm{NP}}\left(\frac{\nu}{Q}, \kappa\right) = \left[\tilde{f}_{0,\mathrm{NP}}\left(\frac{\nu}{Q}, \kappa\right)\right]^{1/(1-a)} \quad , \tag{22}$$

which should be experimentally testable without great effort using existing LEP data. Lacking a direct experimental analysis, the scaling rule was tested against the output of `PYTHIA`, with positive results [11, 26]. The physical picture underlying the scaling rule is appealing, and once again reminiscent of the "tube model". The relevant feature of the radiation pattern is boost invariance, which applies to soft gluons emitted in the two-jet limit, since such emissions are correctly represented by the eikonal approximation. This means that soft gluons contribute to the event shape a rapidity-independent amount, and in turn the integration over rapidity simply measures the size of the region where gluons can be emitted without strongly affecting the event shape. This region scales with $(1-a)^{-1}$. The derivation of the scaling rule neglects correlations between gluons emitted into the opposite hemispheres defined by the thrust axis, which however are expected to become important only for $a \geq 1$, the region excluded by the present treatment. The main assumption is then that nonperturbative soft radiation should share the property of boost inveriance with the relatively harder perturbative component which is treated by resummation. This is the nonperturbative property that would be directly tested by an experimental study of Eq. (22).

One can go further, and study subleading power corrections, mostly related to radiation collinear to the hard jets. Applying the same method to the anomalous dimension B, and to the subleading terms generated by A, once again one finds a simple pattern: the Laplace transform of the cross section can be expressed in factorized form, introducing a subleading shape function, as

$$\tilde{\sigma}\left(\nu, a\right) = \tilde{\sigma}_{\mathrm{PT}}\left(\nu, \kappa, a\right) \, \tilde{f}_{a,\mathrm{NP}}\left(\frac{\nu}{Q}, \kappa\right) \, \tilde{g}_{a,\mathrm{NP}}\left(\frac{\nu}{Q^{2-a}}, \kappa\right) \; . \tag{23}$$

Clearly, as a becomes large and negative, collinear power corrections are expected to become more and more negligible, and the scaling rule Eq. (22) is expected to hold with increasing accuracy.

In the light of the discussion of Sect. (2), it is interesting to verify whether these nice features of angularities are preserved in specific models for the shape function, such as DGE. This is particularly relevant in this case, since the key result, Eq. (22), is related to boost invariance, which is broken by the formal introduction of gluon virtuality, which is a necessary tool of renormalon analysis. The study of angularities with DGE was performed in [12]. It is technically nontrivial, since the interplay of the parameter a with gluon virtuality and phase space constraints makes it difficult to extract the a dependence analytically. The first step is to find an appropriate generalization of the definition of angularity to the case of single massive gluon emission. Such definition should have the correct limit as $\xi \to 0$, reduce to known results for thrust as $a \to 0$, and be simple enough to keep the computational task manageable. At one loop, one such definition is

$$\tau_a = \frac{(1-x_i)^{1-a/2}}{x_i} \left[(1-x_j-\xi)^{1-a/2} (1-x_k+\xi)^{a/2} + (j \leftrightarrow k) \right] , \tag{24}$$

where $x_n = 2p_n \cdot Q/Q^2$ are the customary energy fraction variables, and the definition applies to the phase space region where the gluon is soft; x_i is then the (anti)quark energy fraction. With this definition, it is possible to construct the Borel function for the Sudakov exponent for angularities, in analogy with Eqs. (16) and (17). Remarkably, the soft component of the Borel function, responsible for all leading power corrections, is just the expected rescaling of Eq. (16),

$$B_a^{\rm soft}(\nu, u) = \frac{1}{1-a} \left[2\, {\rm e}^{\kappa u} \, \frac{\sin \pi u}{\pi u} \, \Gamma(-2u) \left(\nu^{2u} - 1 \right) \frac{2}{u} \right] , \tag{25}$$

which leads once again to the scaling rule, Eq. (22). Collinear power corrections are much more difficult to handle analitycally, and the collinear counterpart of Eq. (25) can at best be expressed in terms of a one-dimensional integral representation, which reduces to combinations of hypergeometric functions for rational values of a. This is however enough to classify the singularities in the Borel parameter u, and thus the pattern of power corrections. One indeed finds that all these subleading power corrections can be organized in a single shape function $\tilde{g}$, depending only on the combination ν/Q^{2-a}, as in Eq. (23). DGE thus confirms the general scaling behavior found from resummation, showing that the introduction of gluon virtuality does not spoil the effects of boost invariance in the Sudakov limit. Thrust, jet masses, angularities and the C parameter are all found to have closely related pattern of power corrections, highlighted by the scaling rule relating generic angularities to the thrust. Clearly, this is a highly predictive framework, and our understanding of soft radiation in the two-jet limit can be put to stringent tests.

4. HADRON COLLIDER EVENT SHAPES

As we approach the expected date for the start up of the Large Hadron Collider at CERN, it is natural and appropriate to ask whether tools like those described here could be applicable in the environment of hadron collisions, and, if so, to what extent power corrections might be relevant to our understanding of the data, at the extreme energies available at the LHC. Beginning with the second question, one might naively observe that Λ/Q must be a very small number for any reasonable value of the hard scale Q that one might envisage at the LHC. It would not be wise, however, to neglect power correction studies on this basis, for at least three different reasons.

First of all, it has already been shown at the Tevatron that power corrections have an impact even on observables which are largely dominated by high $p_\perp$ events, and even at very high energy [27]. In Ref. [27], Mangano considered the single-jet inclusive $E_\perp$ distribution, comparing data at different CM energies. One sees that ratios of cross sections at different energies do not have the proper scaling behavior dictated by NLO QCD, but the correct behavior can be recovered by including a power correction determined by a single parameter associated with the normalization of the jet transverse energy. The reason is that even a small shift in the jet $E_\perp$ is amplified in the distribution by the fact that the cross section is falling steeply for increasing $E_\perp$, so that $\delta\sigma/\sigma \sim -n\, \delta E_\perp$ if $\sigma \sim E_\perp^{-n}$. One can expect that in general power corrections will be important for the determination of jet energy scales, and in turn accurate knowledge of these scales may well turn out to be crucial for many high energy studies.

The second reason is that almost any LHC observable will require, before it can be compared with a theoretical prediction, a subtraction of all hadronic activity unrelated to the hard scattering, loosely referred to as "underlying event". There is currently very little, if any, theoretical control on the underlying event, which at the LHC will contain a mix of multiple parton scattering, beam-beam interactions and soft radiation associated with the selected hard process. Even Monte-Carlo methods have difficulties in finding the proper tuning to describe this kind of physics (see, for example, [28]). In this context, the lesson of power correction studies in the gentler environment of e^+e^- annihilation or DIS is that we might learn to discriminate between the different components of soft radiation in hadron collisions. On the one hand there are soft and collinear gluons associated with the hard scattering event: their effects are in principle computable in PQCD, using generalizations of the known techniques, and their distribution in phase space and in the space of color configurations will be nontrivial and predictable. On the other hand, there is soft radiation which is in practice out of reach for the techniques of PQCD, such as minijets due to multiple parton scattering or soft gluons arising from beam-beam interactions. This second kind of radiation fills phase space with a high degree of uniformity, and will have to be modelled with different techniques, including Monte-Carlo tools. This kind of statistical modelling is bound to be more successful if we can first achieve a better understanding of the "pure" hard scattering process, including the energy and color flow that it generates at all scales.

Finally, it should be emphasized that the physics of event shapes at hadron colliders is interesting for its own sake, as a probe to understand hadronization in terms of both momentum and color flow. It is well known that for hadron collisions, where most processes involve four partons already at Born level, Sudakov resummation is expressed in terms of anomalous dimension matrices that tie together color exchange and momentum flow. This interplay is bound to influence the pattern of power corrections as well, and studies of this kind may well lead to deep insights into the mechanics of color neutralization and hadron formation.

Preliminary studies of resummed event shapes at hadron colliders have already been performed [29, 30], and a generalization of the notion of angularity to a hadronic environment has been proposed [31]. For most of these proposals, a primary concern is that of suppressing the contributions of particles close to the beam axis, where beam remnants interfere with all measurements. It should be noted, however, that soft radiation associated with the underlying event tends to fill phase space, including regions separated in rapidity from both the beam and the high-$p_\perp$ jets. It would therefore be of great interest to find event shapes designed to focus on this kind of wide-angle radiation, where it would be most useful to disentangle soft gluons generated by the hard scattering from the genuine underlying event (a pioneering study with a similar goal is [32]). A promising avenue of investigation might be the use of observables such as those introduced in [23], joint distributions correlating energy flow in a chosen angular region Ω with a standard event shape such as ordinary angularity. The form of such a correlation is

$$\sigma\left(\epsilon_1, \epsilon_2, a\right) = \frac{1}{2s} \sum_N \overline{|M(N)|^2} \; \delta(\epsilon_1 - f_\Omega(N)) \; \delta(\epsilon_2 - \tau_a^{(1)}(N) - \tau_a^{(2)}(N)) \; , \tag{26}$$

where $f_\Omega(N)$ is an observable related to energy flow into the angular region Ω, away from hard jets, while $\tau_a^{(i)}(N)$ are the contributions to angularity from the two hemispheres defined by the thrust axis.

In a hadronic environment, one might envisage measuring angularities with respect to the current jet axis, as suggested in [31], or even introducing a third parameter ϵ_3 to constrain radiation near the beam remnants. Tuning the various parameters and energy threshold in such observables one would be able to focus on different regions of phase space, forcing the observable to be more or less inclusive for soft gluons, or for particles collinear either to the beam or to the current jets. Notice that boost invariance, underlying the scaling rule for angularities, is lost in hadron collisions, where correlations between beam jets and current jets must be taken into account.

Clearly, event shape studies at hadron colliders are in their early days. I believe that such studies will be both instrumental to further our understanding of QCD, and very helpful in order to exploit the full potential of the LHC, a task which will require a solid understanding of strong interactions as much as good skills in the building and testing of new physics models.

Acknowledgments

I would like to thank Carola Berger and Einan Gardi, who worked with me on the research described here, and went over the draft of this contribution, spotting several misprints and one significant inaccuracy, now fixed. I also thank George Sterman and Mrinal Dasgupta for sharing their insights, and the organizers and FRIF for the opportunity granted by the Workshop, which was conducted in a friendly and constructive atmosphere. This work was supported in part by MIUR under contract 2004021808_009.

References

[1] J. C. Collins, D. E. Soper and G. Sterman, *Adv. Ser. Direct. High Energy Phys.* **5** (1988) 1, `hep-ph/0409313`.
[2] M. Beneke, V.M. Braun and L. Magnea, *Nucl. Phys.* **B 497** (1997) 297, `hep-ph/9701309`.
[3] M. Beneke, *Phys. Rept.* **317** (1999) 1, `hep-ph/9807443`.
[4] E. Laenen, *Pramana* **63** (2004) 1225.
[5] E. Gardi, *Nucl. Phys.* **B 622** (2002) 365, `hep-ph/0108222`.
[6] M. Dasgupta and G.P. Salam, *J. Phys.* **G 30** (2004) R143, `hep-ph/0312283`.
[7] L. Magnea, in *Proceedings* of IFAE 2002, SIF, ed. M Cacciari *et. al*, p. 143, `hep-ph/0211013`.
[8] E. Gardi and J. Rathsman, *Nucl. Phys.* **B 609** (2001) 123, `hep-ph/0103217`.
[9] E. Gardi and J. Rathsman, *Nucl. Phys.* **B 638** (2002) 243, `hep-ph/0201019`.
[10] E. Gardi and L. Magnea, *JHEP* **0308** (2003) 030, `hep-ph/0306094`.
[11] C.F. Berger and G. Sterman, *JHEP* **0309** (2003) 058, `hep-ph/0307394`.
[12] C. F. Berger and L. Magnea, *Phys. Rev.* **D 70** (2004) 094010, `hep-ph/0407024`.
[13] S. Catani, L. Trentadue, G. Turnock and B.R. Webber, *Nucl. Phys.* **B 407** (1993) 3.
[14] G.P. Korchemsky and G. Sterman, *Nucl. Phys.* **B 555** (1999) 335, `hep-ph/9902341`.
[15] E. Gardi, these proceedings, `hep-ph/0606080`.
[16] B. R. Webber, `hep-ph/9411384`.
[17] G.P. Korchemsky and S. Tafat, *JHEP* **0010** (2000) 010, `hep-ph/0007005`.
[18] A.V. Belitsky, G.P. Korchemsky and G. Sterman, *Phys. Lett.* **B 515** (2001) 297, `hep-ph/0106308`.
[19] P. Ball, M. Beneke and V.M. Braun, *Nucl. Phys.* **B 452** (1995) 563, `hep-ph/9502300`.
[20] Y.L. Dokshitzer, G. Marchesini and B.R. Webber, *Nucl. Phys.* **B 469** (1996) 93, `hep-ph/9512336`.
[21] S. Catani, B. R. Webber and G. Marchesini, *Nucl. Phys.* **B 349** (1991) 635.
[22] S. Catani and B.R. Webber, *Phys. Lett.* **B 427** (1998) 377, `hep-ph/9801350`.
[23] C.F. Berger, T. Kucs and G. Sterman, *Phys. Rev.* **D 68** (2003) 014012, `hep-ph/0303051`.
[24] M. Dasgupta and G. P. Salam, *Phys. Lett.* **B 512** (2001) 323, `hep-ph/0104277`.
[25] Y. L. Dokshitzer, A. Lucenti, G. Marchesini and G. P. Salam, *JHEP* **9801** (1998) 011, `hep-ph/9801324`.
[26] C.F. Berger and G. Sterman, `hep-ph/0310058`.
[27] M. L. Mangano, `hep-ph/9911256`.
[28] S. Alekhin *et al.*, `hep-ph/0601012`.
[29] A. Banfi, G. P. Salam and G. Zanderighi, *JHEP* **0408** (2004) 062, `hep-ph/0407287`.
[30] G. Zanderighi, these proceedings, `hep-ph/0605332`.
[31] C. F. Berger, *Mod. Phys. Lett.* **A 20** (2005) 1187, `hep-ph/0505037`.
[32] G. Marchesini and B. R. Webber, *Phys. Rev.* **D 38** (1988) 3419.